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Remarks on Symmetric Bi derivations of Rings

Moharram A. Khan

A-4/970, Skyline Appartments , Dodhpur Civil lines, Aligarh-202002 (U.P.), India
E-mail: nassb@hotmail.com

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Abstract. The aim of the paper is to improve the results in [1] which is as follows: Let R be a semiprime ring of characteristic not a prime number m which admits a symmetric bi derivation B such that $[[[B(x; x); x]; x]; x] = 0$ holds for all $x \in R$: Then $[B(x; x); x] = 0$, for all $x \in R$. Finally, some commutativity results are also investigated.

Keywords: Centers, Commutator identities, Semiprime rings, Symmetric bi derivations

1. Introduction

Throughout, R represents an associative ring with centre $Z(R)$. Recall that R is prime if aRb = f0g implies $a = 0$ or $b = 0$, and R is semiprime if aRa = f0g implies $a = 0$. As usual, we write $[x; y]$ for xy ¡ yx and use basic commutator identities $[xy; z] = [x; z]y + x[y; z]$ and $[x; yz] = y[x; z] + [x; y]z$. An additive mapping d : R ¡! R is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x; y$ 2 R. A mapping G : R ¡! R is said to be centralizing on R if $[G(x); x]$ 2 $Z(R)$ for all x 2 R. In particular, if $[G(x); x] = 0$, for all x 2 R, then the mapping G is called commuting on R. A mapping B(¢; ¢) : R £ R ¡! R will be called symmetric if $B(x; y) = B(y; x)$ for all pairs $x; y$ 2 R. A mapping g : R ¡! R de¯ned by $g(x) = B(x; x)$, where B(¢; ¢) : R£R ¡! R is a symmetric mapping will be called the trace of B. Obviously, if B(¢; ¢) : R £ R ¡! R is a symmetric mapping which is also bi additive (i.e. additive in both arguments), then the trace of B satisfies the relation $g(x + y) = g(x) + g(y) + 2B(x; y)$ for all $x; y \in R$.